

## Chapter 11: Equations

Equation 11.1:

$$y_i = \alpha + \beta_1 x_{1i} + \beta_2 x_{2i} + \varepsilon_i$$

Equation 11.2:

$$y_i = a_{yx_1} + b_{yx_1} x_{1i} + e_{(yx_1)i}$$

Equation 11.3:

$$b_{yx_1} = \frac{\sum_{i=1}^n (x_{1i} - \bar{x}_1) y_i}{\sum_{i=1}^n (x_{1i} - \bar{x}_1) x_{1i}}$$

Equation 11.4:

$$E(b_{yx_1}) = \frac{\sum_{i=1}^n (x_{1i} - \bar{x}_1) E(y_i)}{\sum_{i=1}^n (x_{1i} - \bar{x}_1) x_{1i}}$$

Equation 11.5:

$$E(y_i) = \alpha + \beta_1 x_{1i} + \beta_2 x_{2i}$$

Equation 11.6:

$$E(b_{yx_1}) = \beta_1 + \beta_2 \frac{\sum_{i=1}^n (x_{1i} - \bar{x}_1) x_{2i}}{\sum_{i=1}^n (x_{1i} - \bar{x}_1) x_{1i}}$$

Equation 11.7:

$$y_i = -20,000 + 4,000x_{1i} + 350x_{2i} + \varepsilon_i$$

Equation 11.8:

$$y_i = -11,589 + 4,591x_i + e_i$$

Equation 11.9:

$$\beta_2 \frac{\sum_{i=1}^n (x_{1i} - \bar{x}_1) x_{2i}}{\sum_{i=1}^n (x_{1i} - \bar{x}_1) x_{1i}}$$

Equation 11.10:

$$\frac{\sum_{i=1}^n (x_{1i} - \bar{x}_1) x_{2i}}{\sum_{i=1}^n (x_{1i} - \bar{x}_1) x_{1i}}$$

Equation 11.11:

$$x_{2i} = a_{x_2x_1} + b_{x_2x_1} x_{1i} + e_{(x_2x_1)i}$$

Equation 11.12:

$$y_i = a + b_1x_{1i} + b_2x_{2i} + e_i$$

Equation 11.13:

$$\hat{y}_i = a + b_1x_{1i} + b_2x_{2i}$$

Equation 11.14:

$$e_i = y_i - \hat{y}_i = y_i - (a + b_1x_{1i} + b_2x_{2i})$$

Equation 11.15:

$$\sum_{i=1}^n e_i^2 = \sum_{i=1}^n (y_i - (a + b_1x_{1i} + b_2x_{2i}))^2$$

Equation 11.16:

$$\frac{\partial \left( \sum_{i=1}^n e_i^2 \right)}{\partial a} = -2 \sum_{i=1}^n (y_i - (a + b_1x_{1i} + b_2x_{2i}))$$

Equation 11.17:

$$0 = -2 \sum_{i=1}^n (y_i - (a + b_1x_{1i} + b_2x_{2i}))$$

Equation 11.18:

$$0 = \sum_{i=1}^n (y_i - (a + b_1 x_{1i} + b_2 x_{2i}))$$

Equation 11.19:

$$0 = \sum_{i=1}^n (y_i - (a + b_1 x_{1i} + b_2 x_{2i})) = \sum_{i=1}^n (y_i - \hat{y}_i) = \sum_{i=1}^n e_i$$

Equation 11.20:

$$\frac{\partial \left( \sum_{i=1}^n e_i^2 \right)}{\partial b_1} = -2 \sum_{i=1}^n (y_i - (a + b_1 x_{1i} + b_2 x_{2i})) x_{1i}$$

Equation 11.21:

$$0 = -2 \sum_{i=1}^n (y_i - (a + b_1 x_{1i} + b_2 x_{2i})) x_{1i}$$

Equation 11.22:

$$0 = \sum_{i=1}^n (y_i - (a + b_1 x_{1i} + b_2 x_{2i})) x_{1i}$$

Equation 11.23:

$$0 = \sum_{i=1}^n (y_i - (a + b_1 x_{1i} + b_2 x_{2i})) x_{1i} = \sum_{i=1}^n (y_i - \hat{y}_i) x_{1i} = \sum_{i=1}^n e_i x_{1i}$$

Equation 11.24:

$$0 = \frac{\sum_{i=1}^n (e_i - \bar{e})(x_{1i} - \bar{x}_1)}{n-1} = \text{COV}(e_i, x_{1i})$$

Equation 11.25:

$$0 = \sum_{i=1}^n (y_i - (a + b_1 x_{1i} + b_2 x_{2i})) x_{2i}$$

Equation 11.26:

$$0 = \sum_{i=1}^n (y_i - (a + b_1 x_{1i} + b_2 x_{2i})) x_{2i} = \sum_{i=1}^n (y_i - \hat{y}_i) x_{2i} = \sum_{i=1}^n e_i x_{2i}$$

Equation 11.27:

$$0 = \frac{\sum_{i=1}^n (e_i - \bar{e})(x_{2i} - \bar{x}_2)}{n-1} = \text{COV}(e_i, x_{2i})$$

Equation 11.28:

$$a = \bar{y} - b_1 \bar{x}_1 - b_2 \bar{x}_2$$

Equation 11.29:

$$\bar{y} = a + b_1 \bar{x}_1 + b_2 \bar{x}_2$$

Equation 11.30:

$$0 = \sum_{i=1}^n (y_i - \bar{y})x_{1i} - b_1 \sum_{i=1}^n (x_{1i} - \bar{x}_1)x_{1i} - b_2 \sum_{i=1}^n (x_{2i} - \bar{x}_2)x_{1i}$$

Equation 11.31:

$$0 = \sum_{i=1}^n (y_i - \bar{y})x_{2i} - b_1 \sum_{i=1}^n (x_{1i} - \bar{x}_1)x_{2i} - b_2 \sum_{i=1}^n (x_{2i} - \bar{x}_2)x_{2i}$$

Equation 11.32:

$$b_2 = \frac{\sum_{i=1}^n (y_i - \bar{y})x_{2i} - b_1 \sum_{i=1}^n (x_{1i} - \bar{x}_1)x_{2i}}{\sum_{i=1}^n (x_{2i} - \bar{x}_2)x_{2i}}$$

Equation 11.33:

$$0 = \sum_{i=1}^n (y_i - \bar{y})x_{1i} - b_1 \sum_{i=1}^n (x_{1i} - \bar{x}_1)x_{1i} - \left( \frac{\sum_{i=1}^n (y_i - \bar{y})x_{2i} - b_1 \sum_{i=1}^n (x_{1i} - \bar{x}_1)x_{2i}}{\sum_{i=1}^n (x_{2i} - \bar{x}_2)x_{2i}} \right) \sum_{i=1}^n (x_{2i} - \bar{x}_2)x_{1i}$$

Equation 11.34:

$$\begin{aligned}
 0 = & \sum_{i=1}^n (y_i - \bar{y})x_{1i} - b_1 \sum_{i=1}^n (x_{1i} - \bar{x}_1)x_{1i} \\
 & - \frac{\sum_{i=1}^n (y_i - \bar{y})x_{2i} \sum_{i=1}^n (x_{2i} - \bar{x}_2)x_{1i}}{\sum_{i=1}^n (x_{2i} - \bar{x}_2)x_{2i}} \\
 & + \frac{b_1 \sum_{i=1}^n (x_{1i} - \bar{x}_1)x_{2i} \sum_{i=1}^n (x_{2i} - \bar{x}_2)x_{1i}}{\sum_{i=1}^n (x_{2i} - \bar{x}_2)x_{2i}}
 \end{aligned}$$

Equation 11.35:

$$\begin{aligned}
 0 = & \sum_{i=1}^n (y_i - \bar{y})x_{1i} - \frac{\sum_{i=1}^n (y_i - \bar{y})x_{2i} \sum_{i=1}^n (x_{2i} - \bar{x}_2)x_{1i}}{\sum_{i=1}^n (x_{2i} - \bar{x}_2)x_{2i}} \\
 & - b_1 \left( \sum_{i=1}^n (x_{1i} - \bar{x}_1)x_{1i} - \frac{\sum_{i=1}^n (x_{1i} - \bar{x}_1)x_{2i} \sum_{i=1}^n (x_{2i} - \bar{x}_2)x_{1i}}{\sum_{i=1}^n (x_{2i} - \bar{x}_2)x_{2i}} \right)
 \end{aligned}$$

Equation 11.36:

$$\begin{aligned}
 & b_1 \left( \sum_{i=1}^n (x_{1i} - \bar{x}_1)x_{1i} - \frac{\sum_{i=1}^n (x_{1i} - \bar{x}_1)x_{2i} \sum_{i=1}^n (x_{2i} - \bar{x}_2)x_{1i}}{\sum_{i=1}^n (x_{2i} - \bar{x}_2)x_{2i}} \right) \\
 & = \sum_{i=1}^n (y_i - \bar{y})x_{1i} - \frac{\sum_{i=1}^n (y_i - \bar{y})x_{2i} \sum_{i=1}^n (x_{2i} - \bar{x}_2)x_{1i}}{\sum_{i=1}^n (x_{2i} - \bar{x}_2)x_{2i}}
 \end{aligned}$$

Equation 11.37:

$$b_1 = \frac{\sum_{i=1}^n (y_i - \bar{y})x_{1i} - \frac{\sum_{i=1}^n (y_i - \bar{y})x_{2i} \sum_{i=1}^n (x_{2i} - \bar{x}_2)x_{1i}}{\sum_{i=1}^n (x_{2i} - \bar{x}_2)x_{2i}}}{\sum_{i=1}^n (x_{1i} - \bar{x}_1)x_{1i} - \frac{\sum_{i=1}^n (x_{1i} - \bar{x}_1)x_{2i} \sum_{i=1}^n (x_{2i} - \bar{x}_2)x_{1i}}{\sum_{i=1}^n (x_{2i} - \bar{x}_2)x_{2i}}}$$

Equation 11.38:

$$b_2 = \frac{\sum_{i=1}^n (y_i - \bar{y})x_{2i} - \frac{\sum_{i=1}^n (y_i - \bar{y})x_{1i} \sum_{i=1}^n (x_{1i} - \bar{x}_1)x_{2i}}{\sum_{i=1}^n (x_{1i} - \bar{x}_1)x_{1i}}}{\sum_{i=1}^n (x_{2i} - \bar{x}_2)x_{2i} - \frac{\sum_{i=1}^n (x_{2i} - \bar{x}_2)x_{1i} \sum_{i=1}^n (x_{1i} - \bar{x}_1)x_{2i}}{\sum_{i=1}^n (x_{1i} - \bar{x}_1)x_{1i}}}$$

Equation 11.39:

$$y_i = -26,242 + 4,063x_{1i} + 474.0x_{2i} + e_i$$

Equation 11.40:

$$\text{earnings} = -17,394 + 3,817(\text{years of schooling}) + e_i$$

Equation 11.41:

$$\text{GNI per capita} = -7,399 + 4,013(\text{CPI}) + e_i$$



Equation 11.42:

$$\text{GNI per capita} = -21,216 + 3,603(\text{CPI}) + 17.42(\text{EDI}) + e_i$$

Equation 11.43:

$$\text{child mortality} = 200.5 - 1.764 \left( \begin{array}{l} \text{percentage of rural population with} \\ \text{access to improved water} \end{array} \right) + e_i$$

Equation 11.44:

$$\text{child mortality} = 189.3 - 1.505 \left( \begin{array}{l} \text{percentage of rural population with} \\ \text{access to improved water} \end{array} \right) - .00175(\text{GNI}) + e_i$$

Equation 11.45:

$$\frac{\sum_{i=1}^n (y_i - \bar{y}) x_{2i} \sum_{i=1}^n (x_{2i} - \bar{x}_2) x_{1i}}{\sum_{i=1}^n (x_{2i} - \bar{x}_2) x_{2i}}$$

Equation 11.46:

$$\frac{\sum_{i=1}^n (x_{1i} - \bar{x}_1) x_{2i} \sum_{i=1}^n (x_{2i} - \bar{x}_2) x_{1i}}{\sum_{i=1}^n (x_{2i} - \bar{x}_2) x_{2i}}$$

Equation 11.47:

$$\sum_{i=1}^n (x_{1i} - \bar{x}_1) x_{1i} = \sum_{i=1}^n (x_{1i} - \bar{x}_1)^2$$

Equation 11.48:

$$\sum_{i=1}^n (x_{2i} - \bar{x}_2) x_{2i} = \sum_{i=1}^n (x_{2i} - \bar{x}_2)^2$$

Equation 11.49:

$$\sum_{i=1}^n (x_{1i} - \bar{x}_1) x_{2i} = \sum_{i=1}^n (x_{2i} - \bar{x}_2) x_{1i} = \sum_{i=1}^n (x_{1i} - \bar{x}_1)(x_{2i} - \bar{x}_2)$$

Equation 11.50:

$$\sum_{i=1}^n (y_i - \bar{y}) x_{1i} = \sum_{i=1}^n (y_i - \bar{y})(x_{1i} - \bar{x}_1)$$

Equation 11.51:

$$\sum_{i=1}^n (y_i - \bar{y}) x_{2i} = \sum_{i=1}^n (y_i - \bar{y})(x_{2i} - \bar{x}_2)$$

Equation 11.52:

$$b_1 = \frac{\sum_{i=1}^n (y_i - \bar{y})(x_{1i} - \bar{x}_1) - \frac{\sum_{i=1}^n (y_i - \bar{y})(x_{2i} - \bar{x}_2) \sum_{i=1}^n (x_{2i} - \bar{x}_2)(x_{1i} - \bar{x}_1)}{\sum_{i=1}^n (x_{2i} - \bar{x}_2)^2}}{\sum_{i=1}^n (x_{1i} - \bar{x}_1)^2 - \frac{\sum_{i=1}^n (x_{1i} - \bar{x}_1)(x_{2i} - \bar{x}_2) \sum_{i=1}^n (x_{2i} - \bar{x}_2)(x_{1i} - \bar{x}_1)}{\sum_{i=1}^n (x_{2i} - \bar{x}_2)^2}}$$

Equation 11.53:

$$x_{1i} = a_{x_1x_2} + b_{x_1x_2} x_{2i} + e_{(x_1x_2)i}$$

Equation 11.54:

$$e_{(x_1x_2)i} = (x_{1i} - \bar{x}_1) - b_{x_1x_2} (x_{2i} - \bar{x}_2)$$

Equation 11.55:

$$\sum_{i=1}^n e_{(x_1x_2)i}^2 = \sum_{i=1}^n (x_{1i} - \bar{x}_1)^2 - b_{x_1x_2}^2 \sum_{i=1}^n (x_{2i} - \bar{x}_2)^2$$

Equation 11.56:

$$b_{x_1x_2} = \frac{\sum_{i=1}^n (x_{2i} - \bar{x}_2)(x_{1i} - \bar{x}_1)}{\sum_{i=1}^n (x_{2i} - \bar{x}_2)^2}$$

Equation 11.57:

$$\begin{aligned} \sum_{i=1}^n e_{(x_1x_2)i}^2 &= \sum_{i=1}^n (x_{1i} - \bar{x})^2 - \left( \frac{\sum_{i=1}^n (x_{2i} - \bar{x}_2)(x_{1i} - \bar{x}_1)}{\sum_{i=1}^n (x_{2i} - \bar{x}_2)^2} \right)^2 \sum_{i=1}^n (x_{2i} - \bar{x}_2)^2 \\ &= \sum_{i=1}^n (x_{1i} - \bar{x})^2 - \frac{\sum_{i=1}^n (x_{2i} - \bar{x}_2)(x_{1i} - \bar{x}_1) \sum_{i=1}^n (x_{2i} - \bar{x}_2)(x_{1i} - \bar{x}_1)}{\sum_{i=1}^n (x_{2i} - \bar{x}_2)^2} \end{aligned}$$

Equation 11.58:

$$\sum_{i=1}^n e_{(x_1, x_2)i}^2 = \sum_{i=1}^n \left( e_{(x_1, x_2)i} - \bar{e}_{(x_1, x_2)} \right)^2$$

Equation 11.59:

$$y_i = a_{yx_2} + b_{yx_2} x_{2i} + e_{(yx_2)i}$$

Equation 11.60:

$$e_{(yx_2)i} = (y_i - \bar{y}) - b_{yx_2} (x_{2i} - \bar{x}_2)$$

Equation 11.61:

$$\begin{aligned} \sum_{i=1}^n e_{(x_1, x_2)i} e_{(yx_2)i} &= \sum_{i=1}^n (x_{1i} - \bar{x}_1)(y_i - \bar{y}) \\ &\quad - \frac{\sum_{i=1}^n (x_{2i} - \bar{x}_2)(y_i - \bar{y})}{\sum_{i=1}^n (x_{2i} - \bar{x}_2)^2} \sum_{i=1}^n (x_{1i} - \bar{x}_1)(x_{2i} - \bar{x}_2) \\ &\quad - \frac{\sum_{i=1}^n (x_{2i} - \bar{x}_2)(x_{1i} - \bar{x}_1)}{\sum_{i=1}^n (x_{2i} - \bar{x}_2)^2} \sum_{i=1}^n (x_{2i} - \bar{x}_2)(y_i - \bar{y}) \\ &\quad + \frac{\sum_{i=1}^n (x_{2i} - \bar{x}_2)(x_{1i} - \bar{x}_1)}{\sum_{i=1}^n (x_{2i} - \bar{x}_2)^2} \frac{\sum_{i=1}^n (x_{2i} - \bar{x}_2)(y_i - \bar{y})}{\sum_{i=1}^n (x_{2i} - \bar{x}_2)^2} \sum_{i=1}^n (x_{2i} - \bar{x}_2)^2 \end{aligned}$$

Equation 11.62:

$$\sum_{i=1}^n e_{(x_1, x_2)i} e_{(y, x_2)i} = \sum_{i=1}^n (x_{1i} - \bar{x}_1)(y_i - \bar{y}) - \frac{\sum_{i=1}^n (x_{2i} - \bar{x}_2)(y_i - \bar{y}) \sum_{i=1}^n (x_{1i} - \bar{x}_1)(x_{2i} - \bar{x}_2)}{\sum_{i=1}^n (x_{2i} - \bar{x}_2)^2}$$

Equation 11.63:

$$\sum_{i=1}^n e_{(x_1, x_2)i} e_{(y, x_2)i} = \sum_{i=1}^n (e_{(x_1, x_2)i} - \bar{e}_{(x_1, x_2)}) (e_{(y, x_2)i} - \bar{e}_{(y, x_2)})$$

Equation 11.64:

$$b_1 = \frac{\sum_{i=1}^n e_{(x_1, x_2)i} e_{(y, x_2)i}}{\sum_{i=1}^n e_{(x_1, x_2)i}^2}$$

Equation 11.65:

$$b_1 = \frac{\sum_{i=1}^n (e_{(x_1, x_2)i} - \bar{e}_{(x_1, x_2)}) (e_{(y, x_2)i} - \bar{e}_{(y, x_2)})}{\sum_{i=1}^n (e_{(x_1, x_2)i} - \bar{e}_{(x_1, x_2)})^2}$$

Equation 11.66:

$$e_{(y, x_2)i} = \text{intercept} + b e_{(x_1, x_2)i} + \text{residual}$$

Equation 11.67:

$$\text{earnings} = -30,664 + 3,708(\text{years of schooling}) + 364.5(\text{age}) + e_i$$

Equation 11.68:

$$\text{years of schooling} = 11.39 + .02806(\text{age}) + e_i$$

Equation 11.69:

$$\text{earnings} = 11,561 + 468.6(\text{age}) + e_i$$

Equation 11.70:

$$\text{residual of earnings} = .0000 + 3,708(\text{residual of schooling}) + e_i$$

Equation 11.71:

$$\sum_{i=1}^n (y_i - \bar{y}) x_{1i} = \sum_{i=1}^n (x_{1i} - \bar{x}_1) y_i$$

Equation 11.72:

$$\sum_{i=1}^n (y_i - \bar{y}) x_{2i} = \sum_{i=1}^n (x_{2i} - \bar{x}_2) y_i$$

Equation 11.73:

$$\sum_{i=1}^n (x_{1i} - \bar{x}_1) x_{2i} = \sum_{i=1}^n (x_{2i} - \bar{x}_2) x_{1i}$$

Equation 11.74:

$$b_1 = \frac{\sum_{i=1}^n (x_{1i} - \bar{x}_1) y_i - \frac{\sum_{i=1}^n (x_{2i} - \bar{x}_2) y_i \sum_{i=1}^n (x_{2i} - \bar{x}_2) x_{1i}}{\sum_{i=1}^n (x_{2i} - \bar{x}_2) x_{2i}}}{\sum_{i=1}^n (x_{1i} - \bar{x}_1) x_{1i} - \frac{\sum_{i=1}^n (x_{2i} - \bar{x}_2) x_{1i} \sum_{i=1}^n (x_{2i} - \bar{x}_2) x_{1i}}{\sum_{i=1}^n (x_{2i} - \bar{x}_2) x_{2i}}}$$

Equation 11.75:

$$b_1 = \frac{\sum_{i=1}^n (x_{1i} - \bar{x}_1) y_i - \frac{\sum_{i=1}^n (x_{2i} - \bar{x}_2) x_{1i}}{\sum_{i=1}^n (x_{2i} - \bar{x}_2) x_{2i}} \sum_{i=1}^n (x_{2i} - \bar{x}_2) y_i}{\sum_{i=1}^n (x_{1i} - \bar{x}_1) x_{1i} - \frac{\left( \sum_{i=1}^n (x_{2i} - \bar{x}_2) x_{1i} \right)^2}{\sum_{i=1}^n (x_{2i} - \bar{x}_2) x_{2i}}}$$

Equation 11.76:

$$\frac{\sum_{i=1}^n (x_{2i} - \bar{x}_2) x_{1i}}{\sum_{i=1}^n (x_{2i} - \bar{x}_2) x_{2i}}$$

Equation 11.77:

$$\frac{\sum_{i=1}^n (x_{2i} - \bar{x}_2) x_{1i}}{\sum_{i=1}^n (x_{2i} - \bar{x}_2) x_{2i}} \sum_{i=1}^n (x_{2i} - \bar{x}_2) y_i = \sum_{i=1}^n \left( \frac{\sum_{i=1}^n (x_{2i} - \bar{x}_2) x_{1i}}{\sum_{i=1}^n (x_{2i} - \bar{x}_2) x_{2i}} \right) (x_{2i} - \bar{x}_2) y_i$$

Equation 11.78:

$$b_1 = \frac{\sum_{i=1}^n \left( (x_{1i} - \bar{x}_1) - \frac{\sum_{i=1}^n (x_{2i} - \bar{x}_2) x_{1i}}{\sum_{i=1}^n (x_{2i} - \bar{x}_2) x_{2i}} (x_{2i} - \bar{x}_2) \right) y_i}{\sum_{i=1}^n (x_{1i} - \bar{x}_1) x_{1i} - \frac{\left( \sum_{i=1}^n (x_{2i} - \bar{x}_2) x_{1i} \right)^2}{\sum_{i=1}^n (x_{2i} - \bar{x}_2) x_{2i}}}$$

Equation 11.79:

$$E(b_1) = \frac{\sum_{i=1}^n \left( (x_{1i} - \bar{x}_1) - \frac{\sum_{i=1}^n (x_{2i} - \bar{x}_2) x_{1i}}{\sum_{i=1}^n (x_{2i} - \bar{x}_2) x_{2i}} (x_{2i} - \bar{x}_2) \right) (\alpha + \beta_1 x_{1i} + \beta_2 x_{2i})}{\sum_{i=1}^n (x_{1i} - \bar{x}_1) x_{1i} - \frac{\left( \sum_{i=1}^n (x_{2i} - \bar{x}_2) x_{1i} \right)^2}{\sum_{i=1}^n (x_{2i} - \bar{x}_2) x_{2i}}}$$

Equation 11.80:

$$\sum_{i=1}^n \left( (x_{1i} - \bar{x}_1) - \frac{\sum_{i=1}^n (x_{2i} - \bar{x}_2) x_{1i}}{\sum_{i=1}^n (x_{2i} - \bar{x}_2) x_{2i}} (x_{2i} - \bar{x}_2) \right) \alpha$$

Equation 11.81:

$$\alpha \sum_{i=1}^n \left( (x_{1i} - \bar{x}_1) - \frac{\sum_{i=1}^n (x_{2i} - \bar{x}_2) x_{1i}}{\sum_{i=1}^n (x_{2i} - \bar{x}_2) x_{2i}} (x_{2i} - \bar{x}_2) \right)$$



Equation 11.82:

$$\alpha \left( \sum_{i=1}^n (x_{1i} - \bar{x}_1) - \sum_{i=1}^n \frac{\left( \sum_{i=1}^n (x_{2i} - \bar{x}_2) x_{1i} \right)}{\sum_{i=1}^n (x_{2i} - \bar{x}_2) x_{2i}} (x_{2i} - \bar{x}_2) \right)$$

Equation 11.83:

$$\alpha \left( \sum_{i=1}^n (x_{1i} - \bar{x}_1) - \frac{\left( \sum_{i=1}^n (x_{2i} - \bar{x}_2) x_{1i} \right)}{\sum_{i=1}^n (x_{2i} - \bar{x}_2) x_{2i}} \sum_{i=1}^n (x_{2i} - \bar{x}_2) \right)$$

Equation 11.84:

$$\beta_1 \left( \sum_{i=1}^n (x_{1i} - \bar{x}_1) x_{1i} - \frac{\left( \sum_{i=1}^n (x_{2i} - \bar{x}_2) x_{1i} \right)^2}{\sum_{i=1}^n (x_{2i} - \bar{x}_2) x_{2i}} \right)$$

Equation 11.85:

$$E(b_1) = \frac{\beta_1 \left( \sum_{i=1}^n (x_{1i} - \bar{x}_1) x_{1i} - \frac{\left( \sum_{i=1}^n (x_{2i} - \bar{x}_2) x_{1i} \right)^2}{\sum_{i=1}^n (x_{2i} - \bar{x}_2) x_{2i}} \right)}{\sum_{i=1}^n (x_{1i} - \bar{x}_1) x_{1i} - \frac{\left( \sum_{i=1}^n (x_{2i} - \bar{x}_2) x_{1i} \right)^2}{\sum_{i=1}^n (x_{2i} - \bar{x}_2) x_{2i}}} = \beta_1$$